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Cosmic strings interacting with dark strings

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ABSTRACT: Motivated by astrophysical observations of excess electronic production in the galaxy, new theoretical models of the dark matter sector have been proposed in which the Standard model couples to the dark matter sector through an attractive interaction. The coupling of the Standard model to “dark strings”, which are solutions of the low energy dark sector has been investigated recently. Here, we discuss the interaction between dark strings and standard cosmic strings and show that they can form bound states. In the presence of the interaction term, a Bogomolny-Prasad-Sommerfield (BPS) bound exists that depends on the interaction parameter and we observe that the attractive interaction between dark strings and cosmic strings is most efficient if the two strings are identical. Moreover, our model allows for dark string solution that can lower their energy by coupling to an electromagnetic field. We also investigate the gravitational properties of these solutions and show that they become supermassive with a singular space-time for values of the gravitational coupling larger as compared to the non-interacting case. Moreover, the deficit angle of the solutions decreases with increasing interaction.

KEYWORDS: Topological Strings, Spontaneous Symmetry Breaking

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1 Introduction

There is strong observational evidence [1] that approximately 22% of the total energy density of the universe is in the form of dark matter. Up until now it is unclear what this dark matter should be made of. One of the favourite candidates are Weakly Interacting Massive Particles (WIMPs) which arise in extensions of the Standard Model. Recently, new theoretical models of the dark matter sector have been proposed [2], in which the Standard Model is coupled to the dark sector via an attractive interaction term. These models have been motivated by new astrophysical observations [3] which show an excess in electronic production in the galaxy. Depending on the experiment, the energy of these excess electrons is between a few GeV and a few TeV. One possible explanation for these observations is the annihilation of dark matter into electrons. Below the GeV scale, the interaction term in these models is basically of the form of a direct coupling between the U(1) field strength tensor of the dark matter sector and the U(1) field strength tensor of electromagnetism. The U(1) symmetry of the dark sector has to be spontaneously broken, otherwise a “dark photon” background leading to observable consequences would exist.

Consequently, it has been shown that the dark sector can have string-like solutions, denominated “dark strings” and the observational consequences of the interaction of these dark strings with the Standard Model have been discussed [4].

Topological defects are believed to have formed in the numerous phase transitions in the early universe due to the Kibble mechanism [5]. While magnetic monopoles and

domain walls, which result from the spontaneous symmetry breaking of a spherical and parity symmetry, respectively, are catastrophic for the universe since they would overclose it, cosmic strings are an acceptable remnant from the early universe. These objects form whenever an axial symmetry gets spontaneously broken and (due to topological arguments) are either infinitely long or exist in the form of cosmic string loops. Numerical simulations of the evolution of cosmic string networks have shown that these reach a scaling solution, i.e. their contribution to the total energy density of the universe becomes constant at some stage. The main mechanism that allows cosmic string networks to reach this scaling solution is the formation of cosmic string loops due to self-intersection and the consequent decay of these loops under the emission of gravitational radiation.

For some time, cosmic strings were believed to be responsible for the structure formation in the universe. New Cosmic Microwave background (CMB) data, however, clearly shows that the theoretical power spectrum associated to Cosmic strings is in stark contrast to the observed power spectrum. However, there has been a recent revival of cosmic strings since it is now believed that cosmic strings might be linked to the fundamental strings of string theory [6].

While perturbative fundamental strings were excluded to be observable on cosmic scales for many reasons [7], there are now new theories containing extra dimensions, so-called brane world model, that allow to lower the fundamental Planck scale down to the TeV scale. This and the observation that cosmic strings generically form at the end of inflation in inflationary models resulting from String Theory [8] and Supersymmetric Grand Unified Theories [9] has boosted the interest in cosmic string solutions again. The interaction of cosmic strings has been investigated in the context of field theoretical models describing bound systems of D- and F-strings, so-called p-q-strings [10, 11].

Here we study the interaction of cosmic strings with dark string solutions. On a field theoretical level, the model is similar to the one used in [10, 11], however in this paper, the interaction between the strings is mediated via the gauge fields (and gravity), while the strings in [10, 11] interact via a potential (and gravity). While the dark sector in our model is an Abelian Higgs model, we should in principle couple the corresponding solutions to the electromagnetic field of the Standard model. Here, we assume the U(1) symmetry to be spontaneously broken and the corresponding Abelian-Higgs model to possess cosmic string solutions. In fact, we employ the strategy that the Standard model (and its semilocal brother) have string-like solutions [12] which share many features with the solutions of the U(1) toy model. Solutions in this U(1) Abelian-Higgs model have been first discussed in [13]. These solutions have a magnetic field with a quantized magnetic flux and are topological solitons in the sense that they have a topological charge associated to them. When the gauge boson mass is equal to the Higgs boson mass in this theory, the string solutions fulfill an energy bound, the Bogomolny-Prasad-Sommerfield (BPS) bound [14], such that the energy per unit length is directly proportional to the topological charge. These solutions have been discussed extensively and their gravitational properties have also been investigated [15–17]. The main feature of the space-time around a cosmic string is that it is locally flat, but globally possesses a deficit angle δ that is directly proportional to the energy per unit length of the solutions μ : $\delta = 8\pi G\mu$, where G is Newton's constant.

This leads e.g. to gravitational lensing effects that should make it possible to detect cosmic strings in the universe. Interestingly, globally regular gravitating strings exist only as long as the solutions are not too massive. If they become too massive (or the gravitational coupling becomes too large) the deficit angle is larger than 2π and the space-time is singular [18]. It has been noted [16, 17] that solutions that are BPS in flat space-time are also BPS in curved space-time, i.e. fulfill the same energy bound.

Our paper is organized as follows: in section 2, we give the model, the equations and discuss the BPS bound. In section 3, we present our numerical results and section 4 contains our conclusions and an outlook.

2 The model

We study the interaction of a U(1) Abelian-Higgs field model, which has cosmic string solutions, with the low energy dark sector, which is also a U(1) Abelian-Higgs model in flat and curved space-time, respectively. Note that the U(1) model is a toy model here for standard model-like theories with gauge group $SU(2) \times U(1)$, which also contain string solutions. Examples would be semilocal strings in the $SU(2)_{\text{global}} \times U(1)$ model and electroweak strings in the $SU(2)_{\text{local}} \times U(1)$ model [12], respectively. We believe that our toy model captures the qualitative features of these theories.

The model we are studying is given by the following action:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} R + \mathcal{L}_m \right) \tag{2.1}$$

where R is the Ricci scalar and G denotes Newton's constant. The matter Lagrangian \mathcal{L}_m reads:

$$\mathcal{L}_m = D_\mu \phi (D^\mu \phi)^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \xi (D^\mu \xi)^* - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - V(\phi, \xi) + \frac{\varepsilon}{2} F_{\mu\nu} H^{\mu\nu} \tag{2.2}$$

with the covariant derivatives $D_\mu \phi = \nabla_\mu \phi - ie_1 A_\mu \phi$, $D_\mu \xi = \nabla_\mu \xi - ie_2 a_\mu \xi$ and the field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $H_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ of the two U(1) gauge potential A_μ , a_μ with coupling constants e_1 and e_2 . ϕ and ξ are complex scalar fields (Higgs fields) with potential

$$V(\phi, \xi) = \frac{\lambda_1}{4} (\phi\phi^* - \eta_1^2)^2 + \frac{\lambda_2}{4} (\xi\xi^* - \eta_2^2)^2 \tag{2.3}$$

The term proportional to ε is the interaction term [4]. To be compatible with observations, ε should be on the order of 10^{-3} .

In the following, we associate the dark strings to the fields A_μ and ϕ , while the standard cosmic strings are described by the fields a_μ and ξ . The Higgs fields have masses $M_{H,i} = \sqrt{\lambda_i} \eta_i$, while the gauge boson masses are $M_{W,i} = \sqrt{2} e_i \eta_i$, $i = 1, 2$.

2.1 The Ansatz

In the following we shall analyse the system of coupled differential equations associated with the gravitationally coupled system described above. This system will contain the Euler-Lagrange equations for the matter fields and the Einstein equations for the metric

fields. In order to do that, let us write down the matter and gravitational fields as shown below. The most general, cylindrically symmetric line element invariant under boosts along the z -direction is:

$$ds^2 = N^2(\rho)dt^2 - d\rho^2 - L^2(\rho)d\varphi^2 - N^2(\rho)dz^2 . \quad (2.4)$$

The non-vanishing components of the Ricci tensor R^ν_μ then read [15]:

$$R^0_0 = -\frac{(LNN')'}{N^2L} , \quad R^\rho_\rho = -\frac{2N''}{N} - \frac{L''}{L} , \quad R^\varphi_\varphi = -\frac{(N^2L)'}{N^2L} , \quad R^z_z = R^0_0 \quad (2.5)$$

where the prime denotes the derivative with respect to ρ .

For the matter and gauge fields, we have [13]:

$$\phi(\rho, \varphi) = \eta_1 h(\rho)e^{in\varphi} , \quad \xi(\rho, \varphi) = \eta_1 f(\rho)e^{im\varphi} \quad (2.6)$$

$$A_\mu dx^\mu = \frac{1}{e_1}(n - P(\rho))d\varphi , \quad a_\mu dx^\mu = \frac{1}{e_2}(m - R(\rho))d\varphi . \quad (2.7)$$

n and m are integers indexing the vorticity of the two Higgs fields around the z -axis.

2.2 Equations of motion

We define the following dimensionless variable and function:

$$x = e_1 \eta_1 \rho , \quad L(x) = \eta_1 e_1 L(\rho) . \quad (2.8)$$

Then, the total Lagrangian $\mathcal{L}_m \rightarrow \mathcal{L}_m/(\eta_1^4 e_1^2)$ depends only on the following dimensionless coupling constants

$$\gamma = 8\pi G \eta_1^2 , \quad \beta_i = \frac{2M_{H,i}^2 \eta_1^2}{M_{W,i}^2 \eta_i^2} = \frac{\lambda_i}{e_i^2} , \quad i = 1, 2 , \quad (2.9)$$

and the dimensionless ratios of the coupling constants and vacuum expectation values, respectively

$$g = \frac{e_2}{e_1} , \quad q = \frac{\eta_2}{\eta_1} . \quad (2.10)$$

Varying the action with respect to the matter fields and metric functions, we obtain a system of six non-linear differential equations. The Euler-Lagrange equations for the matter field functions read:

$$\frac{(N^2 L h)'}{N^2 L} = \frac{P^2 h}{L^2} + \frac{\beta_1}{2}(h^2 - 1)h \quad (2.11)$$

$$\frac{(N^2 L f)'}{N^2 L} = \frac{R^2 f}{L^2} + \frac{\beta_2}{2}(f^2 - q^2)f \quad (2.12)$$

$$(1 - \varepsilon^2) \frac{L}{N^2} \left(\frac{N^2 P'}{L} \right)' = 2h^2 P + 2\varepsilon g R f^2 , \quad (2.13)$$

$$(1 - \varepsilon^2) \frac{L}{N^2} \left(\frac{N^2 R'}{L} \right)' = 2g^2 f^2 R + 2\varepsilon g P h^2 , \quad (2.14)$$

where the prime now and in the following denotes the derivative with respect to x .

We use the Einstein equations in the following form:

$$R_{\mu\nu} = -\gamma \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) , \quad \mu, \nu = t, x, \varphi, z \quad (2.15)$$

where T is the trace of the energy momentum tensor $T = T_{\lambda}^{\lambda}$ and the non-vanishing components of the energy-momentum tensor are (we use the notation of [15]) with $i = 1, 2$:

$$\begin{aligned} T_0^0 &= e_s + e_v + e_w + u , & T_x^x &= -e_s - e_v + e_w + u \\ T_{\varphi}^{\varphi} &= e_s - e_v - e_w + u , & T_z^z &= T_0^0 \end{aligned} \quad (2.16)$$

where

$$e_s = (h')^2 + (f')^2 , \quad e_v = \frac{(P')^2}{2L^2} + \frac{(R')^2}{2g^2L^2} - \frac{\varepsilon R'P'}{g L^2} , \quad e_w = \frac{h^2P^2}{L^2} + \frac{R^2f^2}{L^2} \quad (2.17)$$

and

$$u = \frac{\beta_1}{4} (h^2 - 1)^2 + \frac{\beta_2}{4} (f^2 - q^2)^2 . \quad (2.18)$$

We then obtain

$$\frac{(LNN)'}{N^2L} = \gamma \left[\frac{(P')^2}{2L^2} + \frac{(R')^2}{2g^2L^2} - \frac{\varepsilon R'P'}{g L^2} - u \right] \quad (2.19)$$

and:

$$\frac{(N^2L)'}{N^2L} = -\gamma \left[\frac{2h^2P^2}{L^2} + \frac{2R^2f^2}{L^2} + \frac{(P')^2}{2L^2} + \frac{(R')^2}{2g^2L^2} - \frac{\varepsilon R'P'}{g L^2} + u \right] \quad (2.20)$$

2.3 Boundary conditions

The requirement of regularity at the origin leads to the following boundary conditions:

$$h(0) = 0, \quad f(0) = 0, \quad P(0) = n, \quad R(0) = m \quad (2.21)$$

for the matter fields and

$$N(0) = 1, \quad N'(0) = 0, \quad L(0) = 0, \quad L'(0) = 1 . \quad (2.22)$$

for the metric fields. The finiteness of the energy per unit length requires:

$$h(\infty) = 1, \quad f(\infty) = q, \quad P(\infty) = 0, \quad R(\infty) = 0 . \quad (2.23)$$

2.4 Energy per unit length, magnetic fields and deficit angle

We define as inertial energy per unit length of a solution describing the interaction of a dark string with winding n and a cosmic string with winding m :

$$\mu^{(n,m)} = \int \sqrt{-g_3} T_0^0 dx d\varphi \quad (2.24)$$

where g_3 is the determinant of the 2 + 1-dimensional space-time given by (t, x, φ) . This then reads:

$$\mu^{(n,m)} = 2\pi \int_0^\infty NL(\varepsilon_s + \varepsilon_v + \varepsilon_w + u) dx \quad (2.25)$$

Note that the string tension $T = \int \sqrt{-g_3} T_z^z dx d\varphi$ is equal to the energy per unit length. In flat space-time ($G = 0$) and $\varepsilon = 0$, the energy per unit length of the solution is given by

$$\mu^{(n,m)} = 2\pi\eta_1^2 g_1(\beta_1) + 2\pi\eta_1^2 g_2(\beta_2) \quad (2.26)$$

where g_1 and g_2 are functions that depend only weakly on β_1 and β_2 , respectively with $g_1(2) = 1$ and $g_2(2) = 1$ in the BPS limit. In the following, we will set $\eta_1 = 1$ without loss of generality.

We can define the binding energy of an n -dark string with an m -cosmic string as

$$\mu_{\text{bin}}^{(n,m)} = \mu^{(n,m)} - n\mu^{(1,0)} - m\mu^{(0,1)} \quad (2.27)$$

The magnetic fields associated to the solution can be given when noting that the gauge part of the Lagrangian density can be rewritten as follows [4]:

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{\varepsilon}{2}F_{\mu\nu}H^{\mu\nu} \Rightarrow -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}(1 - \varepsilon^2)H_{\mu\nu}H^{\mu\nu} \quad (2.28)$$

with $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ where $\tilde{A}_\mu = A_\mu - \varepsilon a_\mu$.

The magnetic fields associated to the fields \tilde{A}_μ and a_μ have only a component in z -direction. These components read:

$$\tilde{B}_z(x) = \frac{-P'(x) + \frac{\varepsilon}{g}R'(x)}{e_1 L(x)} \quad \text{and} \quad b_z(x) = -\sqrt{1 - \varepsilon^2} \frac{R'(x)}{e_2 L(x)}, \quad (2.29)$$

respectively. The corresponding magnetic fluxes $\int d^2x B$ are

$$\tilde{\Phi} = \frac{2\pi}{e_1} \left(n - \frac{\varepsilon}{g}m \right) \quad \text{and} \quad \varphi = \sqrt{1 - \varepsilon^2} \frac{2\pi m}{e_2}, \quad (2.30)$$

respectively. Obviously, these magnetic fluxes are not quantized for generic ε .

Finally, the deficit angle $\delta = 8\pi G\mu$ of the solution can be read off directly from the derivative of the metric function $L(x)$. For string-like solutions, the metric functions behave like $N(x \rightarrow \infty) \rightarrow c_1$ and $L(x \rightarrow \infty) \rightarrow c_2 x + c_3$, where c_1, c_2 and c_3 are constants. The deficit angle is then given by:

$$\delta = 2\pi(1 - L'|_{x=\infty}) = 2\pi(1 - c_2). \quad (2.31)$$

2.5 The Bogomolny-Prasad-Sommerfield (BPS) bound

For $\varepsilon = 0$, the model has BPS solutions which satisfy the energy bound $\mu = 2\pi n + 2\pi m$ both in flat [14] as well as in curved space-time [17] for $\beta_1 = 2, \beta_2 = 2$.

Here we will discuss the case $f = h$ ($\beta_1 = \beta_2 \equiv \beta$), $P = R$ ($n = m, g = 1$) for $\varepsilon \neq 0$. In flat space-time, the functions $N \equiv 1$ and $L \equiv x$ and the corresponding BPS equations read

$$h' = \frac{Ph}{x}, \quad (1 - \varepsilon) \frac{P'}{x} = h^2 - 1 \quad (2.32)$$

The solutions then fulfill the energy bound $E = 2\pi n + 2\pi m = 4\pi n$ for

$$\beta = \frac{2}{1 - \varepsilon} \tag{2.33}$$

In curved space-time, i.e. for $\gamma \neq 0$, the above bound is still a BPS bound. The corresponding matter equations read

$$h' = \frac{Ph}{L}, \quad (1 - \varepsilon)\frac{P'}{L} = h^2 - 1. \tag{2.34}$$

We have $e_v = u$ and $e_w = e_s$ such that $T_x^x = T_\varphi^\varphi = 0$. Hence $N(x) \equiv 1$, while L fulfills the following equation:

$$\frac{L''}{L} = -2\gamma \left(\frac{2P^2 h^2}{L^2} + \frac{1}{1 - \varepsilon} (h^2 - 1)^2 \right) \tag{2.35}$$

The deficit angle then reads

$$\delta = 2\pi\gamma(n + m). \tag{2.36}$$

2.6 Dark strings coupled to an unbroken U(1) symmetry

For $\beta_2 = 0$, $q = 0$ (with η_1 finite), we have $f \equiv 0$ and the corresponding U(1) symmetry remains unbroken. For $\varepsilon = 0$, $m = 0$, we would then have $R \equiv 0$. However, for $\varepsilon \neq 0$, $m = 0$ the energy of the solution can be lowered by a non-vanishing derivative of R , i.e. $R' \neq 0$. This corresponds to a non-vanishing magnetic field in z -direction. Due to the attractive nature of the interaction, the dark string can thus lower its energy by coupling to a non-vanishing magnetic field inside the string core. The corresponding boundary conditions for R then read:

$$R(0) = 0 \quad \text{and} \quad R'|_{x=\infty} = 0 \tag{2.37}$$

Note that we don't need R but only R' to vanish at infinity to fulfill the requirement of finiteness of the energy. (2.17) then suggests that the e_v part of the energy density is minimized for $R' = \varepsilon g P'$ inside the dark string core. Using the boundary conditions (2.21), (2.37) we find that $R = \varepsilon g(P - n)$. The energy of an $(n, 0)$ string is thus smaller for $\varepsilon \neq 0$ than for $\varepsilon = 0$. The same is, of course, also true for the inverse case, where $n = 0$ and $m \neq 0$ and R replaced by P in the boundary conditions (2.37).

3 Numerical results

In all our numerical calculations, we have set $q = 1$, $g = 1$. We have solved the set of coupled ordinary differential equations numerically using the ordinary differential equation solver COLSYS [20]. Numerical errors are typically on the order of $10^{-8} - 10^{-9}$.

3.1 $\gamma = 0$

To begin with, we present our results for cosmic-dark string systems in flat space-time $\gamma = 0$. In this case, we have $N \equiv 1$ and $L \equiv x$.

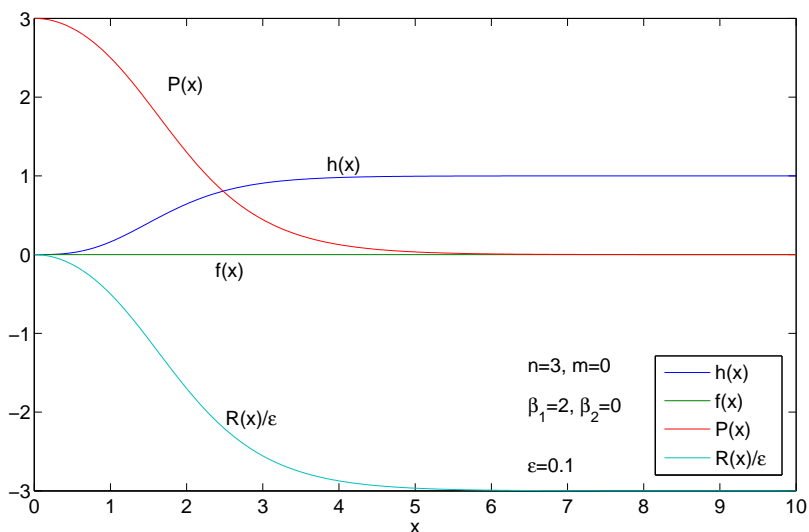


Figure 1. The functions of a dark string solution for $n = 3$, $m = 0$, $\beta_1 = 2$, $\beta_2 = 0$ and $\varepsilon = 0.1$. The fields associated to the dark string are $P(x)$ and $h(x)$, while the scalar field of the unbroken U(1) symmetry vanishes identically $f(x) \equiv 0$. The gauge field function $R(x)$ however is related to the function $P(x)$ by $R(x)/\varepsilon = P(x) - n$.

(n,m)	1	2	3	4	5
1	-0.0004	-0.0006	-0.0008	-0.0009	-0.0009
2	-0.0006	-0.0011	-0.0014	-0.0016	-0.0017
3	-0.0008	-0.0014	-0.0028	-0.0022	-0.0024
4	-0.0009	-0.0016	-0.0022	-0.0026	-0.0030
5	-0.0009	-0.0017	-0.0024	-0.0030	-0.0034

Table 1. The value of the binding energy $\mu_{\text{bin}}^{(n,m)}$ in units of 2π for $\beta_1 = \beta_2 = 2$, $\varepsilon = 0.001$ and different choices of n and m .

We have first studied solutions with $m = 0$, $\beta_2 = 0$. As discussed above, the field $f \equiv 0$, but $R \neq 0$. A typical solutions of this type is shown in figure 1, where we present the functions $P(x)$ and $h(x)$ associated to the dark strings as well as the field $R(x)$ associated to the unbroken U(1) symmetry. The energy per unit length of the dark strings is lowered from $\mu^{(3,0)} = 3$ for $\varepsilon = 0$ to $\mu^{(3,0)} = 2.99080$ for $\varepsilon = 0.1$.

Next, we have investigated the existence of bound states in our system. Following the definition of the binding energy (2.27), we present our results for $\varepsilon \neq 0$ and $\beta_1 = \beta_2 \equiv \beta$. In the $\varepsilon = 0$ limit, $\mu_{\text{bin}}^{(n,m)} > 0$ for $\beta > 2$, $\mu_{\text{bin}}^{(n,m)} < 0$ for $\beta < 2$ and $\mu_{\text{bin}}^{(n,m)} = 0$ for $\beta = 2$ (the BPS limit). This is different here as the tables below suggest. In tables 1,2,3 and 4,5,6 we give the binding energy for $\varepsilon = 0.001$ and $\varepsilon = 0.01$, respectively, for different choices of β and n and m .

Increasing ε increases the attractive interaction between the dark string and the comic string. For $\varepsilon = 0$ and $\beta_1 = \beta_2 = 2$, the strings do not interact, while for the same choice

(n,m)	1	2	3	4	5
1	-0.0005	0.0566	0.1319	0.2169	0.3082
2	0.0566	0.1134	0.1886	0.2734	0.3646
3	0.1319	0.1886	0.2635	0.3482	0.4393
4	0.2169	0.2734	0.3482	0.4328	0.5237
5	0.3082	0.3646	0.4393	0.5237	0.6144

Table 2. The value of the binding energy $\mu_{\text{bin}}^{(n,m)}$ in units of 2π for $\beta_1 = \beta_2 = 3$, $\varepsilon = 0.001$ and different choices of n and m .

(n,m)	1	2	3	4	5
1	-0.0005	0.1049	0.2448	0.4033	0.5738
2	0.1049	0.2099	0.3497	0.5081	0.6784
3	0.2448	0.3497	0.4893	0.6475	0.8177
4	0.4033	0.5081	0.6475	0.8055	0.9755
5	0.5738	0.6784	0.8177	0.9755	1.1454

Table 3. The value of the binding energy $\mu_{\text{bin}}^{(n,m)}$ in units of 2π for $\beta_1 = \beta_2 = 4$, $\varepsilon = 0.001$ and different choices of n and m .

(n,m)	1	2	3	4	5
1	-0.0042	-0.0064	-0.007	-0.0085	-0.0090
2	-0.0064	-0.0109	-0.0138	-0.0158	-0.0171
3	-0.0077	-0.0138	-0.0183	-0.0216	-0.0240
4	-0.0085	-0.0158	-0.0216	-0.0262	-0.0297
5	-0.0090	-0.0171	-0.0240	-0.0297	-0.0344

Table 4. The value of the binding energy $\mu_{\text{bin}}^{(n,m)}$ in units of 2π for $\beta_1 = \beta_2 = 2$, $\varepsilon = 0.01$ and different choices of n and m .

(n,m)	1	2	3	4	5
1	-0.0046	0.0501	0.1240	0.2081	0.2988
2	0.0501	0.1023	0.1743	0.2571	0.3468
3	0.1240	0.1743	0.2246	0.3258	0.4142
4	0.2081	0.2571	0.3258	0.4055	0.4926
5	0.2988	0.3468	0.4142	0.4926	0.5784

Table 5. The value of the binding energy $\mu_{\text{bin}}^{(n,m)}$ in units of 2π for $\beta_1 = \beta_2 = 3$, $\varepsilon = 0.01$ and different choices of n and m .

of the β s and $\varepsilon \neq 0$ they form bound states for all values of n and m . We observe that the binding becomes stronger with increasing $n + m$. This is related to the fact that the solutions' mass increases with increasing winding such that the binding mechanism is more efficient. Moreover, the binding is stronger for an (n, n) string as compared to an $(n + 1, n - 1)$ string and the binding is more effective for an $(n + 1, n - 1)$ string than for

(n,m)	1	2	3	4	5
1	-0.0049	0.0978	0.2362	0.3936	0.5634
2	0.0978	0.1978	0.3341	0.4900	0.6587
3	0.2362	0.3331	0.4684	0.6226	0.7899
4	0.3936	0.4900	0.6226	0.7752	0.9410
5	0.5634	0.6587	0.7899	0.9410	1.1053

Table 6. The value of the binding energy $\mu_{\text{bin}}^{(n,m)}$ in units of 2π for $\beta_1 = \beta_2 = 4$, $\varepsilon = 0.01$ and different choices of n and m .

an $(n+2, n-2)$ string etc. Apparently, dark strings bind stronger to cosmic strings if they have the same winding.

For $\varepsilon = 0$ and $\beta_1 = \beta_2 = 3$ and $\beta_1 = \beta_2 = 4$, respectively, the strings repel since in this case the radius of the flux tube core is larger than that of the scalar core. Apparently, in this case the attractive nature of the new interaction ($\varepsilon \neq 0$) is not strong enough to overcome the repulsion for these choices of $\beta_1 = \beta_2$, except in the case $n = m = 1$. Apparently, the attractive interaction is just about able to overcome the repulsion that two strings would exert on each other for $\varepsilon = 0$ and $\beta_i > 2$, $i = 1, 2$.

We have next determined the critical value of $\beta_1 = \beta_2 \equiv \beta = \beta_{\text{cr}}$ for which the transition between bound and unbound strings takes place. For $\varepsilon = 0$, this happens at $\beta_{\text{cr}}(\varepsilon = 0) = 2$ such that for $\beta < 2$, the binding energy is negative, while for $\beta > 2$, the binding energy is positive. Here, the additional attractive interaction allows us to increase the ratio between Higgs and gauge boson mass to values larger than $\beta = 2$ before the dark string and cosmic string become unbound. To understand the dependence of β_{cr} on ε , we have chosen solutions with $n + m$ constant and have determined the corresponding β_{cr} for $\varepsilon = 0.01$. Our results are given in figure 2.

We plot the mass $\mu^{(n,m)}$ of solutions with $(n, m) = (4 + k, 4 - k)$, $k = 0, 1, 2, 3$. For comparison, we also plot $(4 + k)$ times the mass of an $(1, 0)$ solution plus $(4 - k)$ times the mass of an $(0, 1)$ solution. At the intersection points of this latter curve with the $(4 + k, 4 - k)$ curves, we have $\mu_{\text{bin}}^{(n,m)} = 0$. For all solutions, we observe that the mass increases linearly with $\beta_1 = \beta_2 \equiv \beta$ in the range $\beta \in [2 : 2.1]$ and that the mass of an $(4 + k_1, 4 - k_1)$ solution is higher than that of a $(4 + k_2, 4 - k_2)$ solution for $k_1 > k_2$. Moreover, β_{cr} , i.e. the value at which the strings become unbound is largest for the solution with $n = 4$, $m = 4$ and decreases for k increasing. Again, this results from the fact that dark strings “bind best” to cosmic strings which have the same winding.

The magnetic fields (2.29) change as compared to the $\varepsilon = 0$ case. We observe that $\tilde{B}_z(0)$ decreases with increasing ε , while $b_z(0)$ increases with increasing ε . In addition the core radii of both flux tubes decrease with increasing ε .

3.2 $\gamma \neq 0$

We have first studied the dependence of the deficit angle δ on the interaction parameter ε . Our results are shown in figure 3 for $\gamma = 0.2$, $\beta_1 = \beta_2 = 2$ and different choices of n and m . For $\varepsilon = 0$, the deficit angle $\delta(\varepsilon = 0)$ is given by (2.36). Here, we plot the

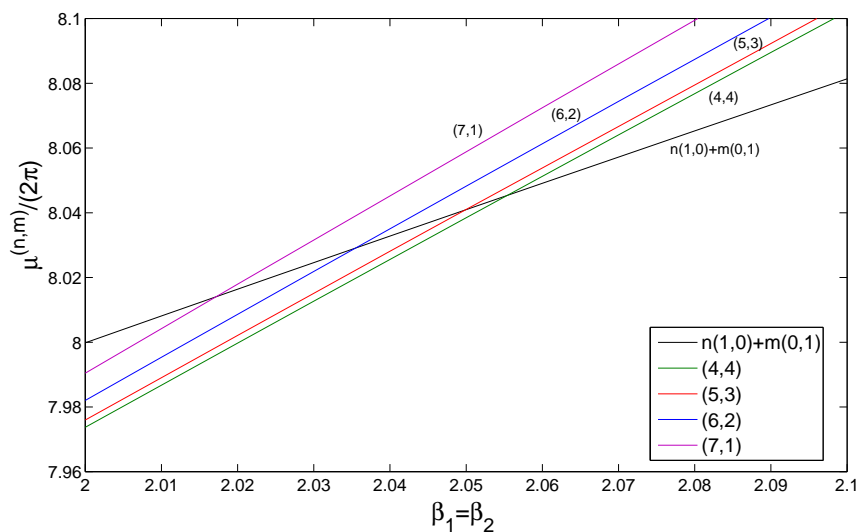


Figure 2. The value of the inertial mass $\mu^{(n,m)}$ in units of 2π is given in dependence on $\beta_1 = \beta_2$ for $\epsilon = 0.01$ and different choices of n and m . Note that $n + m = 8$. For comparison, we also plot n times the mass of the $(1, 0)$ solution plus m times the mass of the $(0, 1)$ solution. At the intersection of the (n, m) curves with this latter curve, we have $\mu_{\text{bin}}^{(n,m)} = 0$.

difference between $\delta(\epsilon = 0)$ and $\delta(\epsilon \neq 0)$. The deficit angle decreases with increasing ϵ and apparently decreases (approximately) linearly with ϵ . The larger the sum $n + m$, the stronger δ decreases with ϵ , which is related to the fact that the higher $n + m$, the more massive the solutions are and the more effective is the attractive interaction between the dark string and the cosmic string. Moreover, for a fixed sum $n + m$, the deficit angle decreases stronger for an (n, n) string as compared to an $(n + 1, n - 1)$ string. The reason for this is that the binding mechanism is better for a dark string and a cosmic string with equal winding. This has also been observed before in models describing interacting cosmic strings when the interaction is mediated via a potential term [10, 11].

Another important feature of cosmic strings in a curved space-time is that globally regular gravitating solutions exist only up to a maximal value of the gravitational coupling $\gamma = \gamma_{\text{max}}(\epsilon)$. For $\gamma > \gamma_{\text{max}}$, the deficit angle becomes larger than 2π , i.e. the solutions become singular. For $\epsilon = 0$, these solutions have been denominated “supermassive” string solutions [18]. For $\epsilon = 0$ and $\beta_1 = \beta_2 = 2$ we have $\gamma_{\text{max}}(\epsilon = 0) = 1/(n + m)$. For $\epsilon > 0$, the solutions exist on a larger interval of γ . We find e.g. for $n = 1, m = 1$ and $\beta_1 = \beta_2 = 2$ that $\gamma_{\text{max}}(\epsilon = 0.01) \approx 0.501$, while $\gamma_{\text{max}}(\epsilon = 0.1) \approx 0.515$. This is not surprising since the total energy per unit length of the solutions decreases for increasing ϵ . Since the deficit angle is proportional to the product of the gravitational coupling and the energy per unit length, the gravitational coupling can be increased stronger before δ becomes equal to 2π .

For $n \neq 0$ and $m = 0$, we find the same phenomenon as in flat space-time: dark string solutions can lower their energy by coupling to the gauge field of an unbroken $U(1)$ symmetry. In figure 4, we compare the masses of string-like solutions for $n = 1, m = 1$ and $n = 2, m = 0$, respectively, for $\gamma = 0.5$. For $n = 1, m = 1$, we have $\beta_1 = \beta_2 = 2$, while for

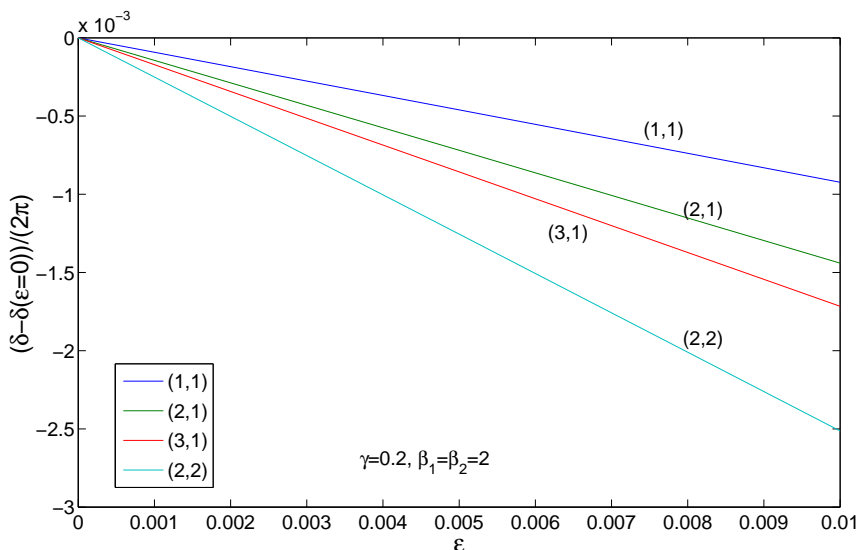


Figure 3. The difference between the deficit angle $\delta/(2\pi)$ and the deficit angle for $\varepsilon = 0$, $\delta(\varepsilon = 0)/(2\pi)$ is given as function of ε for $\gamma = 0.2$, $\beta_1 = \beta_2 = 2$ and different choices of (n, m) .

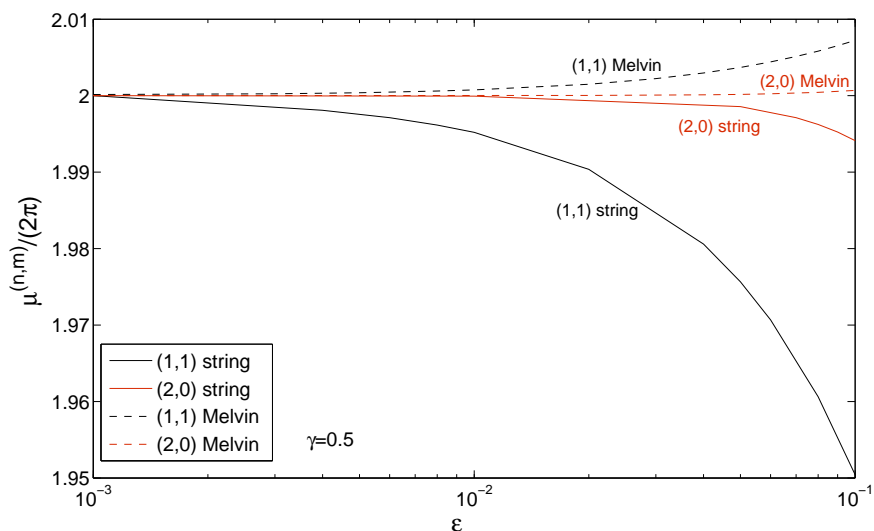


Figure 4. The value of the inertial mass $\mu^{(n,m)}$ of string and Melvin solutions is shown in dependence on the interaction parameter ε and $\gamma = 0.5$. For $n = m = 1$, we have chosen $\beta_1 = \beta_2 = 2$, while for $n = 2, m = 0$, we have $\beta_1 = 2, \beta_2 = 0$.

$n = 2, m = 0$, we have $\beta_1 = 2, \beta_2 = 0$. In both cases, the mass of the solution is decreasing with increasing interaction. It is thus apparent that also in curved space-time, a dark string can lower its energy when coupling to an unbroken $U(1)$ gauge symmetry. Moreover, the total energy of the $(1, 1)$ system is always lower than that of the $(2, 0)$ system. Hence, the binding is more efficient if a dark string and a cosmic string couple than if a dark string couples to a pure gauge field.

It has been observed that string-like solutions with a behaviour of the metric functions $N(x \rightarrow \infty) \rightarrow c_1$ and $L(x \rightarrow \infty) \rightarrow c_2x + c_3$ have a “shadow solution” in the form of so-called Melvin solutions [15, 16]. In contrast to string-like solutions, Melvin solutions have no flat space-time counterparts and their metric functions behave as $N(x \rightarrow \infty) \rightarrow a_1x^{2/3}$ and $L(x \rightarrow \infty) \rightarrow a_2x^{-1/3}$, where a_1 and a_2 are constants. These solutions are also present in systems where cosmic strings interact via a potential term [11] and we also find them here. In figure 4, we give the inertial mass of these solutions for two different choices of n and m . When comparing to the string solutions, it is apparent that the mass of the Melvin solutions is higher for all choices of the interaction parameter. Moreover, in contrast to string solutions, the inertial mass of the Melvin solutions increases for increasing interaction. This has also been observed in [11], where two cosmic strings interact via a potential. The stronger the attractive interaction between the strings, the higher the inertial mass of the Melvin solutions. In addition, the mass increases stronger for an (1, 1) system of dark strings than for an $n = 2$ dark string coupled to the gauge field of the unbroken U(1) symmetry. This is related to the observation that the binding is stronger for the (1, 1) system than for the (2, 0) system.

4 Conclusions and outlook

In this paper, we have studied the interaction of a dark string with a cosmic string where the interaction is mediated by a coupling term between the field strength tensors of the respective solutions. This type of interaction is motivated by recent models describing the dark matter sector. We observe that a BPS bound exists if the dark string is identical to the cosmic string. In fact, we find that the attractive interaction between the two strings is most efficient in this particular case. In addition, the attractive interaction allows for dark-cosmic strings to exist for larger values of the Higgs to gauge boson ratio. The deficit angle associated to the strings decreases for increasing interaction and globally regular string space-times exist for higher values of the gravitational coupling as compared to the non-interacting case. We also find dark string solutions that can lower their energy by coupling to a U(1) field through the attractive interaction term.

The formation of bound states is of interest for the study of the evolution of string networks. For p-q-strings it has been observed [21] that the formation of bound states leads to an energy loss mechanism that is important in the evolution of string networks towards the scaling regime. If the dark matter sector has indeed dark strings solutions then standard cosmic strings could loose energy by coupling to these strings.

It would also be interesting to understand the influence of the type of interaction studied here on superconducting strings [22]. In its simplest version this would be the coupling of a U(1) \times U(1) model (describing the superconducting string with bosonic charge carriers) coupled to a U(1) model describing the dark string. The field strength tensor of the dark string could then either be coupled to the field strength tensor associated to the broken U(1) symmetry (similar to what has been studied in this paper) or alternatively to the field strength tensor of the unbroken U(1) symmetry describing the carrier field.

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